Q.1.: Discuss the various heat flow processes inside the earth (Marks: 05)

Heat is transported by four main processes: conduction, convection, radiation and advection. Conduction, convection and advection need the materials to transport heat; whereas the radiation can pass heat through space or a vacuum.

Thermal conduction takes place by the transfer of kinetic energy through vibration of electrons and phonons between molecules or atoms, and largely determined by the lattice vibrations. Conduction process is the most significant one for heat transportation in solid materials and thus is very important in the lithosphere, and is neglected in the fluid core, however a significant amount of heat of the core is conducted out along the adiabatic temperature gradient.

Instead the convection process is more dominant in mantle and fluid outer core. This only happens when the real temperature gradient in the mantle or outer core is comparatively more compared with the adiabatic temperature gradient. It is the process where the materials bodily carry the heat by thermal induction. Materials transportation is a cyclic phenomenon in which the fluid parcel is migrated upward when warmed and downward when the heat in the parcel of material drops.

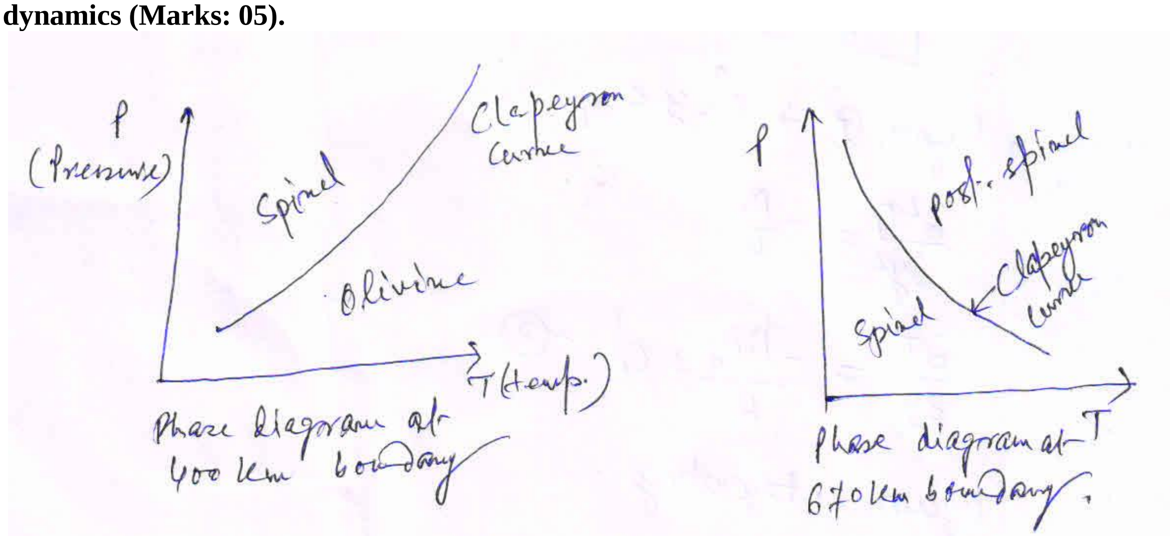
For radiative heat transfer inside the earth, no medium is required, and electromagnetic radiation in the infrared wavelength transfers the heat. However, during heat transfer at high temperature, the conductivity of the materials in the mantle inside the earth is enhanced by extra radiative amount following the formula as given below:

$$K_r = \frac{16}{3}\frac{n^2\sigma}{e}T^3$$

where K_r is the extra increase in conductivity, n is refractive index, σ is Stefan-Boltzmann constant, e is opacity of the medium and T is ambient temperature. The T^3-dependence in the expression illustrates the radiation might be more important than lattice conductivity in the hotter regions of the Earth. However, the effect of increasing temperature is partly offset by an increase in the value of e. Further, the high density of electrons might be absorbing more radiation and raises the opacity, and reduce the heat transfer by radiation in the lower mantle.

Another process of heat transfer is advection, in which the pressure differences facilitate the movement of mass, and thereby carry the heat. It is basically pressure induced forced convection, usually happens with the lava flow during volcanic eruptions, flow of water in thermal spring, etc.

Q.1.(or): Explain the reasons for elevation of the olivine-spinel phase boundary and depression of the spinel–perovskite phase boundary in a subducting oceanic lithosphere. Discuss the signification of such boundaries in mantle dynamics (Marks: 05).



The olivine-spinel boundary is elevated in the descending oceanic lithosphere as compared with its position in the surrounding mantle because the pressure at which the phase change occurs depends on temperature. The left sketch is a plot of the Clapeyron curve, which gives the pressures and temperatures at which two phases of the same material, such as olivine and spinel, are in equilibrium. The slope of the Clapeyron curve γ is given by

$$\gamma = \frac{dp}{dT}$$
$$or, \frac{dz}{dT} = \frac{\gamma}{\rho G}$$

where dp = gρdz. For the olivine to spinel phase change at 400 km depth, the slope of the Clapeyron curve is positive. Since dT is negative for the lower temperatures in the interior of the descending lithosphere, and the olivine-spinel phase change occurs at a shallower depth in the slab. Therefore, the olivine-spinel boundary is elevated inside the descending lithosphere. The elevation of this phase change boundary inside the descending lithosphere increases in density associated, and thereby adding more forces driving the plate downward into the mantle.

Instead, for spinel–perovskite phase boundary (right plot) at 670 km depth, the slope of the Clapeyron curve is negative. Since dT is negative for the lower temperatures in the interior of the descending lithosphere, and the spinel–perovskite phase change occurs at a deeper level in the slab. Therefore, the spinel– perovskite phase boundary is depressed inside the descending lithosphere. The mass deficiency happens inside the descending lithosphere because of the depression of this phase boundary, and the top of the lithosphere is impeded, and suffers back-thrust, which is responsible for the generation of deep focus earthquake. This also opposes the migration of the lower mantle materials into the upper mantle, and allows layered-earth convection.

Q.2. The concentrations of U, Th and K in the mantle of the Earth are 0.025, 0.087 and 70 p.p.m. by weight. Calculate the total heat production per kg inside the mantle by these radioactive nuclides (Marks: 03).

Given that, $C_U = 0.025$ p.p.m

$C_{Th} = 0.87$ p.p.m

$C_K = 70$ p.p.m

formula → Heat production is given by

$$Q_r = (95.2\,C_U + 25.6\,C_{Th} + 0.00348\,C_K)\ \mu W/kg$$

then

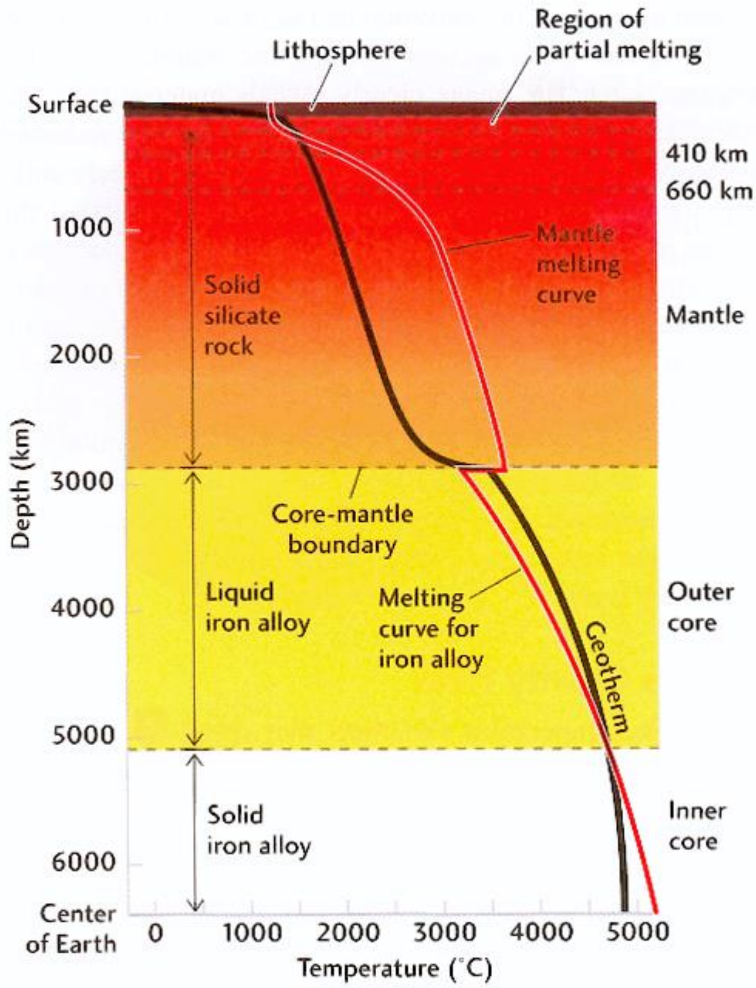
$$Q_r = (95.2\times 0.025\times 10^{-6} + 25.6\times 0.087\times 10^{-6} + 70\times 0.00348\times 10^{-6})\ \mu W/kg$$

$$Q_r = (2.38 + 2.23 + 0.2436)\times 10^{-6}\ \mu W/kg$$

$$= 4.8536\times 10^{-12}\ \mu W/kg$$

Q.3. Explain the variations of (a) temperature, and (b) melting point (solidus) inside the earth with a neat diagram (Marks: 05).

It is apparent from the above plot that the real temperature in the lithosphere is sharply increased, and the melting point is higher in this rigid layer. In the partially melted layer, called asthenosphere, the meting point is little reduced to less than the real temperature. Both increase further beyond the layer of asthenosphere, however the melting point curve increases more sharply up to certain depth (say ~ 1000 km), and both are increased gradually afterwards. Near the lower mantle-outer core boundary, the real temperature increases sharply, and exceeds the values of melting point, which facilitated the transformation of the semi-solid materials into fluid in the outer core. Subsequently, at the outer core-inner core boundary, the melting point increases more sharply, and exceeds the value of real temperature, and the materials inside the core becomes solid.



Q.4. Write short notes on any two of the followings:
i) Critical Rayleigh number, ii) Equilibrium geotherm, iii) Urey's hypothesis for evolution of the Moon, iv) Prandtl Number (Marks: 2×2=4).

i) The Rayleigh number is proportional to the ratio of the buoyancy force to the diffusive-viscous force, and is given by

$$R_a = \frac{g\rho\alpha\Delta T}{\kappa\eta} D^3$$

where Ra is Rayleigh number, g is acceleration due to gravity, α is volume coefficient of expansion, ΔT is the difference in real and adiabatic temperature at a particular depth, D is layer-thickness, κ is thermal diffusivity and η is kinematic viscosity.

Critical Rayleigh number is define as the value at which convection begins in a flat layer of thickness D, when no shear stress on the upper and lower boundaries, and the upper boundary held at a constant temperature while all heating from below and the value of the critical Rayleigh number if 658. The critical Rayleigh number varies under different boundary conditions.

ii) The 3-dimensional heat-conduction equation can be written as

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\nabla^2 T + \frac{A}{\rho c_p}$$

Where T is temperature, t is time, k is thermal conductivity, ρ is density of the materials, c_p is specific heat at constant pressure, A is radioactive heat production.
In one-dimension, where heat-flow is occurring towards the Earth's surface, and temperature increases with depth, the heat-conduction equation is given by

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2} + \frac{A}{\rho c_p}$$

If we consider such one-dimensional column of materials with no erosion or deposition and a constant heat flow, the column may eventually reach a state of thermal equilibrium in which the temperature at any point is steady. In that case, the temperature-depth profile is called an equilibrium geotherm. In this case, $\frac{\partial T}{\partial t} = 0$, and the above equation is reduced to

$$\frac{\partial^2 T}{\partial z^2} = -\frac{A}{k}$$

iii) Urey proposed that the moon had accreted from gas and dust elsewhere in the solar system at its earliest stage, was later captured by the earth at a close distance, and moved out to its present radius from the radius of first capture.
iv) The Prandtl number Pr is a dimensionless number; the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. It is defined as:

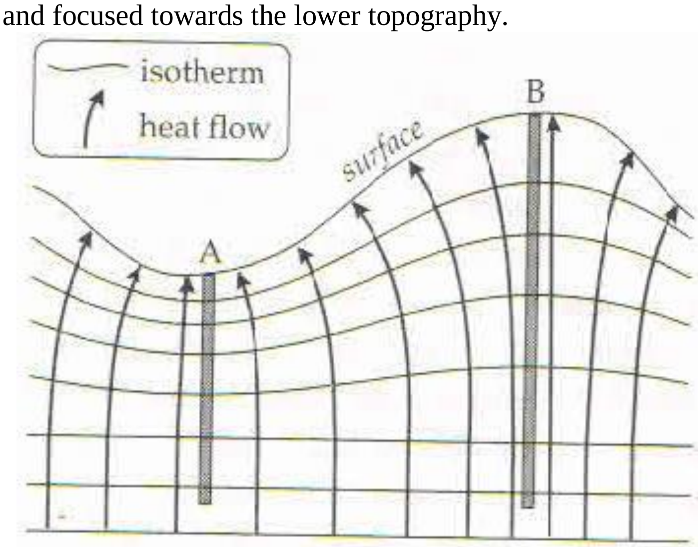
$$P_r = \frac{\nu}{\alpha} = \frac{viscous\ diffusion\ rate}{thermal\ diffusion\ rate}$$

where ν: kinematic viscosity, α: thermal diffusivity

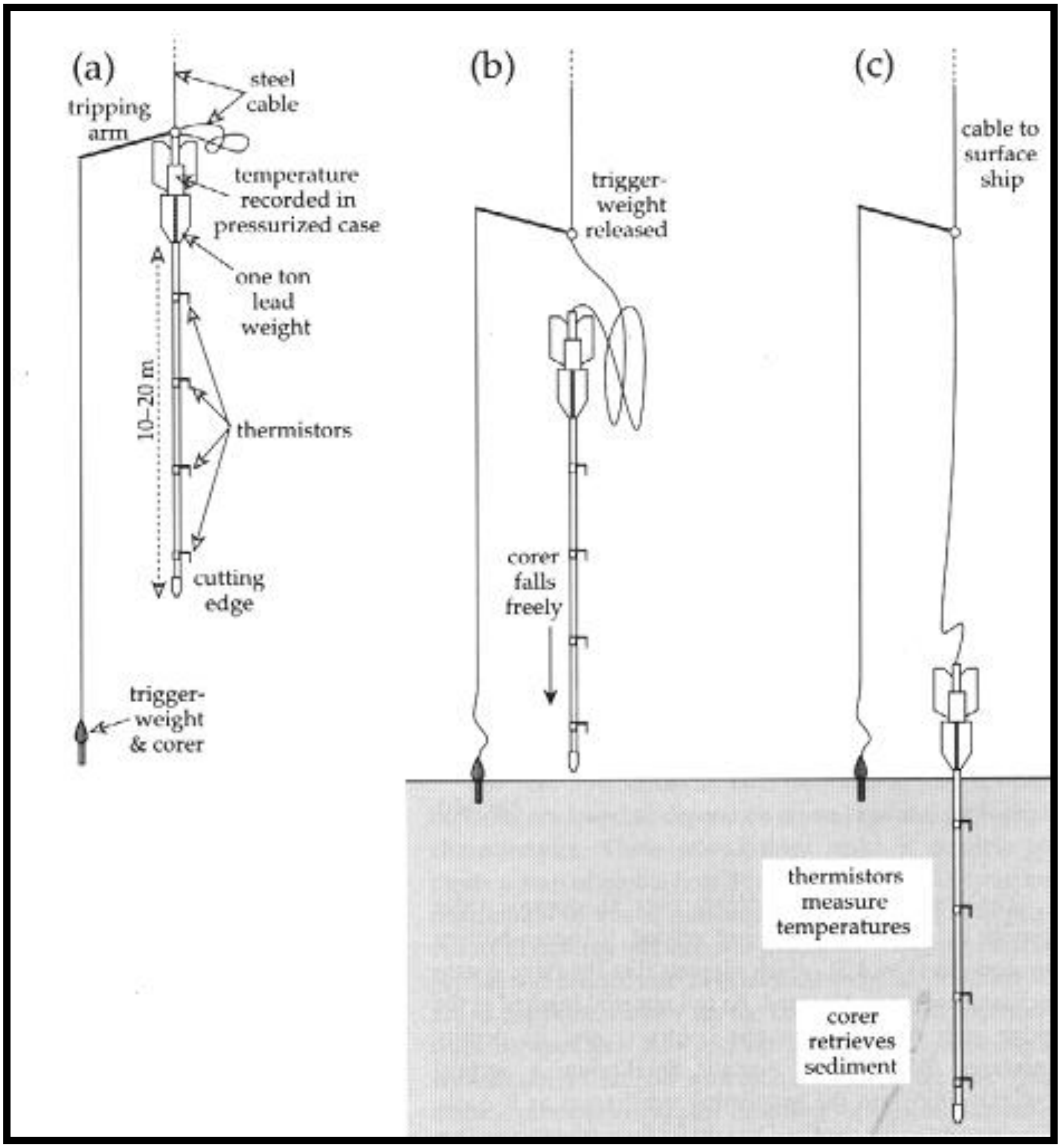
Prandtl number is the physical property of the material and is independent of any flow. For the mantle with $\nu \sim 10^{18}$ m^2/sec and $\kappa \sim 10^{-6}$ m^2/sec, $Pr \sim 10^{24}$, demonstrating that the viscous response to any perturbation is instantaneous compared with the thermal response.

Q.5. Discuss the effect of topography on isotherms and the direction of heat flow with a neat diagram (Marks: 05).

It is generally accepted that the heat-flow is only vertical. Well below the surface the isotherms (surfaces of same temperature) are horizontal and the flow of heat (orthogonal to the isotherms) is vertical. However, with undulating surface (as shown by the figure below), the isotherms follow the topography. Thus, the heat-flow lines are deviated from the higher topography and focused towards the lower topography.



Q.5.(or): Explain a method for measuring the heat flow in the ocean floor with a suitable diagram (Marks: 05).



Heat-flow in the ocean bottom is measured by Ewing piston corer. This device consists of a heavily weighted, hollow sampling pipe, commonly about 10-20 m long as shown in the following figure. A plunger inside the pipe is displaced by sediment during coring and makes a seal with the sediment surface, so that sample loss and core deformation are minimized when the core is withdrawn from the ocean floor. Thermisters are mounted on short arms a few centimeters away from the body of the pipe, and the temperatures are recorded in a water-tight casement.

The instrument is lowered from a surface ship until a free-dangling trigger-weight makes contact with the bottom. This releases the corer, which falls freely and is driven into the sediment by the one-ton lead weight. The sediment-filled corer is hauled back on board the ship, where the thermal conductivity of the sediment can be determined. The estimated thermal conductivity and temperature are used for determination of heat-flow in the ocean bottom.

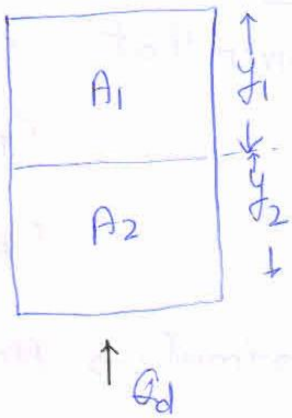
Q.6. Derive the expressions of equilibrium geotherm for two-layered crustal model of the Earth with upper layer of thickness y_1 and radioactive heat generation A_1, and lower layer of thickness y_2 and radioactive heat generation A_2. Assume the heat flow from the mantle to be $Q = 45 mWm^{-2}$ and the temperature $T = 0°C$ at depth $y = 0$-km.

Given that -

Heat flow from mantle

$Q_d = 45\ mWm^2$ and

$T = 0^\circ C$ at $y = 0$



Solution $\Rightarrow$

Heat conduction equation

$$\frac{\partial T}{\partial t} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{A}{\rho C_p}$$

for steady state equilibrium

$$\frac{\partial T}{\partial t} = 0$$

so $$\boxed{\frac{\partial^2 T}{\partial y^2} = -\frac{A}{K}} \quad (1)$$

for layer (I) $\rightarrow$ $0 < y < y_1$

$$\frac{\partial^2 T_1}{\partial y^2} = -\frac{A_1}{K}$$

$$\frac{\partial T_1}{\partial y} = \frac{-A_1 y}{K} + C_1 \quad (2)$$

again integrating

$$T_1 = -\frac{A_1 y^2}{2K} + C_1 y + C_2$$

but at $y=0$ $T=0$

so $\boxed{C_2 = 0}$

then $T_1 = -\frac{A_1 y^2}{2K} + C_1 y$ — (3)

For layer 2 → for $y_1 < y < y_1 + y_2$

$$\frac{\partial^2 T_2}{\partial y^2} = -\frac{A_2}{K}$$

$$\frac{\partial T_2}{\partial y} = -\frac{A_2 y}{K} + C_3 \quad -(4)$$

$$T_2 = -\frac{A_2 y^2}{2K} + C_3 y + C_4 \quad -(5)$$

B from boundary condition - at $y = y_1$

$$\frac{\partial T_1}{\partial y} = \frac{\partial T_2}{\partial y}$$

so from equation (2) and (4)

$$-\frac{A_1 y}{K} + C_1 = -\frac{A_2 y_1}{K} + C_3$$

$$C_1 = \left(\frac{A_1 - A_2}{K}\right) y_1 + C_3$$

putting this value in equation (3)

$$T_1 = -\frac{A_1 y^2}{2K} + \left(\frac{(A_1 - A_2)}{K} y_1 + C_3\right) y \quad -(6)$$

At $y = y_1 + y_2$, $Q = -Q_d$

so from equation (4)

$$\frac{\partial T_2}{\partial y} = \frac{Q_2}{K} = -\frac{A_2 (y_1 + y_2)}{K} + C_3$$

So $C_3 = \frac{Q_2}{K} + \frac{A_2(y_1+y_2)}{K}$

$$T_1 = -\frac{A_1 y^2}{2K} + \left\{\left(\frac{A_1-A_2}{K}\right)y_1 + \frac{Q_2}{K} + \frac{A_2(y_1+y_2)}{K}\right\} y$$

$$T_1 = -\frac{A_1 y^2}{2K} + \left\{\frac{Q_2}{K} + \frac{A_2 y_2}{K} + \frac{A_1 y_1}{K}\right\} y$$

Putting $Q_2 = 45\,mWm^{-2} = 45\times10^{-3}\,Wm^{-2}$

$$\boxed{T_1 = -\frac{A_1 y^2}{2K} + \left\{\frac{45\times10^{-3}}{K} + \frac{A_2 y_2}{K} + \frac{A_1 y_1}{K}\right\} y}$$

Putting C_3 value in equation (5)

$$T_2 = -\frac{A_2 y^2}{2K} + \left(\frac{Q_2}{K} + \frac{A_2(y_1+y_2)}{K}\right)y + C_4 \quad -(7)$$

again from boundary condition
at $y = y_1$ $T_1 = T_2$

$$-\frac{A_1 y_1^2}{2K} + \left\{\frac{Q_2}{K} + \frac{A_2 y_2}{K} + \frac{A_1 y_1}{K}\right\} y_1 = -\frac{A_2 y_1^2}{2K} + \left(\frac{Q_2}{K} + \frac{A_2(y_1+y_2)}{K}\right) y_1 + C_4$$

$$C_4 = \left(\frac{A_1-A_2}{2K}\right) y_1^2 \quad -(8)$$

Putting C_4 value in equation (7)

$$T_2 = -\frac{A_2 y^2}{2K} + \left(\frac{Q_2}{K} + \frac{A_2(y_1+y_2)}{K}\right)y + \left(\frac{A_1-A_2}{2K}\right)y_1^2 \quad (9)$$

putting $Q_2 = 45\times10^{-3}\ Wm^{-2}$

$$\boxed{T_2 = -\frac{A_2 y^2}{2K} + \left(\frac{45\times10^{-3}}{K} + \frac{A_2(y_1+y_2)}{K}\right)y + \left(\frac{A_1-A_2}{2K}\right)y_1^2}$$